

Spatial Math: Visualize, Explore, and Analyze Numbers and Data With Geographic Information Systems (GIS)

Description: Visualize and analyze numbers with this series of 22 lessons, using spatial analysis as your framework and Geographic Information Systems (GIS) as your toolkit.

Software: AEJEE (“Edgy”), ArcExplorer Java Edition for Education, freely available from ESRI, version 2.3.1, is used for all lessons. In addition, selected lessons have been created for ArcGIS Desktop, 9.2 and above. ArcGIS Desktop provides additional functionality while AEJEE provides an easy-to-use introductory tool.

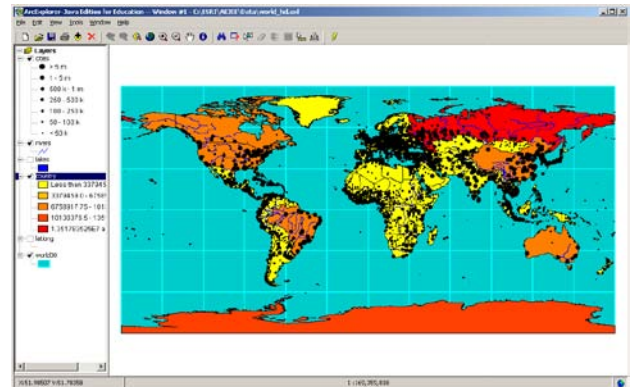
Level: 5 scaffolded lessons for upper elementary (Grades 4-6, ages 9-11), 5 for middle secondary school (Grades 6-8, ages 11-13), and 5 for high secondary school (Grades 9-12, ages 14-18). In addition, 8 high secondary school lessons have been created on data exploration, including 2 on asking questions of your data, 1 on Cartesian coordinate systems, 1 on the geometry of map projections, 1 on ranges and extremes, 1 on distance and travel, 1 on distributions, means, and standard deviations, and 1 on distance=rate x time problems. Each lesson contains 20 questions. Total = 23 lessons x 20 questions = 460 questions.

GIS Skills Required: These lessons assume that no prior knowledge of GIS exists.

Time Required: Each lesson should take 2 one-hour-long class periods, possibly 3 periods in some of the secondary units.

Procedure: 1) Download and install AEJEE software for Windows or Macintosh operating systems computers from <http://edcommunity.esri.com/aejee>. The software comes packaged with the data (maps and tables) needed for these lessons as well as a help file as a file stored under esri\aejee\data\lessons called “aejee23.pdf.”

2) Download the lesson “World Reference” from: <http://edcommunity.esri.com/arclessons/lesson.cfm?id=330>. This zip file contains a small data set for use with AEJEE 2.3.1, showing continents, oceans, and some seas. There is also a project file (an AXL), and a PDF write-up showing where to install files.



A Geographic Information System (GIS) allows you to analyze numbers as tables and maps, visualize spatial patterns, and explore relationships using real-world data and technology.



3) Use one or all of the 23 lessons described in the tables below to teach mathematics skills and concepts with GIS!

For More Information: Visit the Math-GIS education resources at ESRI, on: <http://edcommunity.esri.com/math>.

Summary: Each lesson requires students to analyze data using AEJEE GIS or ArcGIS software. Each lesson provides the steps required to run the software but the emphasis is not on the software but on an inquiry-driven, problem-solving approach. The lessons can be worked on in teams of two or by individual students. Each lesson contains the same 5 themes for each of the 3 age levels, and each lesson contains 20 questions. The lessons can be worked on in any order, but ideally, from 1 to 5—in other words, coordinates first, then measurement, then scale, then classification, and then analyzing data.

Lesson Detail 1 of 3:

Concepts and skills and Maps Used	Upper Primary (Upper Elementary)	Middle Secondary	Upper Secondary
Analyzing Coordinates	Minimum and max differences, latitude and longitude, number lines. World reference map	Minimum and max differences, latitude and longitude, number lines. World reference map	Minimum and max differences, latitude and longitude, number lines. World reference map
Exploring Measurement	Measuring distances and circumferences; units of measure. USA state map	Measuring distances and circumferences; units of measure. USA state map	Measuring distances and circumferences; units of measure. USA state map
Studying Scale	Scales and fractions. World reference map	Scales and fractions. World reference map	Scales and fractions. World reference map
Investigating Classification	Classifying, ranges, symbology. World elevation map	Classifying, ranges, symbology. World elevation map	Classifying, ranges, symbology. World elevation map
Analyzing Data	Sorting, averages, division, units of measure. World map	Sorting, averages, division, units of measure. World map	Sorting, averages, division, units of measure. World map



Lesson Detail 2 of 3:

Concepts and skills	Upper Secondary
Maps Used	
Asking Questions of Your Data I	Spatial patterns, querying, buffers, percentages, sorting, adjacency. World tectonics map.
Asking Questions of Your Data II	Spatial patterns, querying, ranges, units of measurement, buffering, intersections, percentages. World tectonics map.
Exploring Distributions, Means, and Standard Deviations	Mean, standard deviations, distributions, classifications, units of measurement, querying, operators. World tectonics map.
The Land of Cartesia: Exploring the Cartesian Coordinate System and Geometry	Points, lines, polygons, coordinates, measurement, area, perimeter. 10 Grid map, World reference map.
The Geometry of Map Projections	Map projections, angles, shapes, centrality, 2D and 3D. World hemisphere projected map.
Ranges and Extremes	Ranges, extremes, means, patterns World climate map.
Traveling By Land and By Sea	Map projections, measurement. World hemisphere projected map.
Distance = Rate x Time	Map projections, measurement. World hemisphere projected map.



Lesson Detail 3 of 3:

Concepts and skills	Upper Secondary: ArcGIS Version
Maps Used	
Analyzing Coordinates	Minimum and maximum, differences, latitude and longitude, number lines. World reference map
Exploring Measurement	Measuring distances and circumferences; units of measure. USA state map
Studying Scale	Scales and fractions. World reference map
Investigating Classification	Classifying, ranges, symbology. World elevation map
Analyzing Data	Sorting, averages, division, units of measure. World map
The Land of Cartesia: Exploring the Cartesian Coordinate System and Geometry	Points, lines, polygons, coordinates, measurement, area, perimeter. 10 Grid map, World reference map.
Exploring Distributions, Means, and Standard Deviations	Mean, standard deviations, distributions, classifications, units of measurement, querying, operators. World tectonics map.
Distance = Rate x Time	Map projections, measurement. World hemisphere projected map.

For More Information:

- 1) ESRI Math Education Resource Center and discipline landing page: <http://edcommunity.esri.com/math>
- 2) These lessons are stored in the ArcLessons library of GIS-based lessons: <http://edcommunity.esri.com/arclessons>.

For additional information about the use of GIS in education, including software, training, data, and support, visit the ESRI Education Community: <http://edcommunity.esri.com>.



Author: These lessons were authored by the following member of the ESRI education team. Comments and suggestions appreciated!

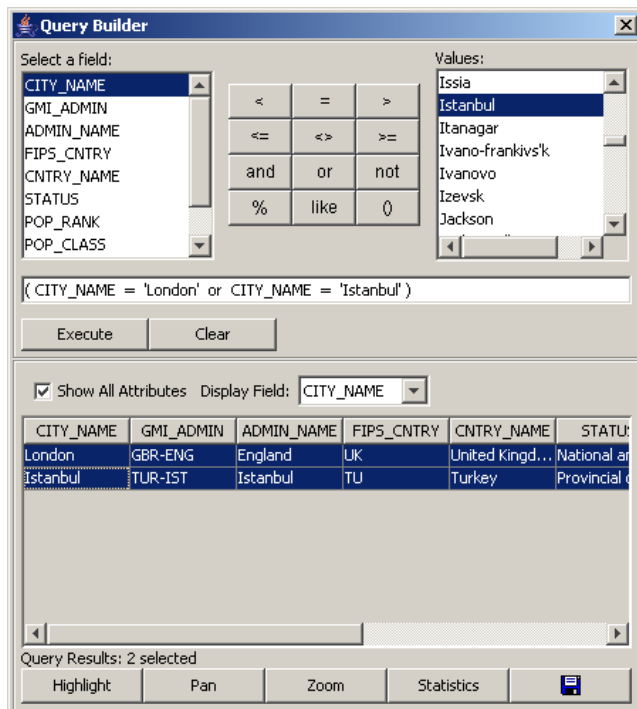
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Sample Questions:



The following questions appear in the lesson “Distance = Rate x Time” and illustrate the integration of spatial and mathematical analysis using technology.

Traveling By Sea

First, you have decided to journey by sea from London, England to Istanbul, Turkey! But now you need to plan to determine how much time you need to ask off of work. To do that, you need to determine the distance between the two cities and calculate how long it will take to reach your final destination, and return.



Use Query  to select these two cities. When your query is made, click on the two cities, and select “highlight”, then “zoom”, as shown at left:

Look at your map. You might need to zoom out  once to see more land and ocean. Use the measure  tool.

1) How far apart are London and Istanbul, in straight-line distance, in km? _____.

2) What direction is Istanbul from London?
_____.



3) If you decided to sail from London to Istanbul by ship due east from London along the Thames River to the North Sea, then southwest along the French coast, then south along the western, Atlantic coast of Europe and through the Straits of Gibraltar into the Mediterranean Sea, describe the route you would need to take, including the list of countries that you will sail past en route from London to Istanbul: _____

_____.

4) How far is your ocean voyage (in km) from London to Istanbul? _____km.

If your ship travels at 25 kph, how long will it take for you to reach Istanbul? Show your work and give the results in hours: _____

_____.

5) How long will it take you to travel **round trip** from London to Istanbul, including a 48 hour stop in Istanbul to see the sights? Show your work with the result in **hours**: _____

_____.

6) How long will it take you to travel round trip from London to Istanbul, including a 48 hour stop in Istanbul? Show your work with the result in **days**:

_____.

7) If your ship uses 1 liter of diesel fuel for every 50 meters, how many liters of diesel fuel will it use for the round trip? Show your work:

_____.

1 liter = 0.26 US gallons.

How many gallons of diesel fuel will your ship use for the round trip? Show your work: _____

_____.

Do some research on fuel efficiency of cruise ships. How realistic is the above fuel efficiency? _____
_____.

8) You have heard that it is also fun to sail on a smaller ship, a 50-person yacht. The yacht is slower than a cruise ship. The yacht you are considering travels at 20 kph and the cruise ship travels at 25 kph. If the yacht that you are considering departs London on Monday morning at 8:00am, and the cruise ship departs London



on Tuesday morning at 8:00am, which will arrive in Istanbul first—the yacht or the cruise ship? Each will follow the ocean route that you identified above. Show your work: _____

If the yacht is passed by the cruise ship, how far from London and Istanbul will this occur? Show your work: _____

If the yacht is passed by the cruise ship, when will this occur? Show your work:

Use measure to determine the following answer: If the yacht is passed by the cruise ship, off of which country's shore will this point occur?

Links To Content Standards:

In 1989, the National Council of Teachers of Mathematics (NCTM) released its Curriculum and Evaluation Standards for School Mathematics. This document provided guidance for developing and implementing a vision of mathematics and instruction that serves all students. In 2000, NCTM expanded and elaborated on the 1989 standards to create *Principles and Standards for School Mathematics*.

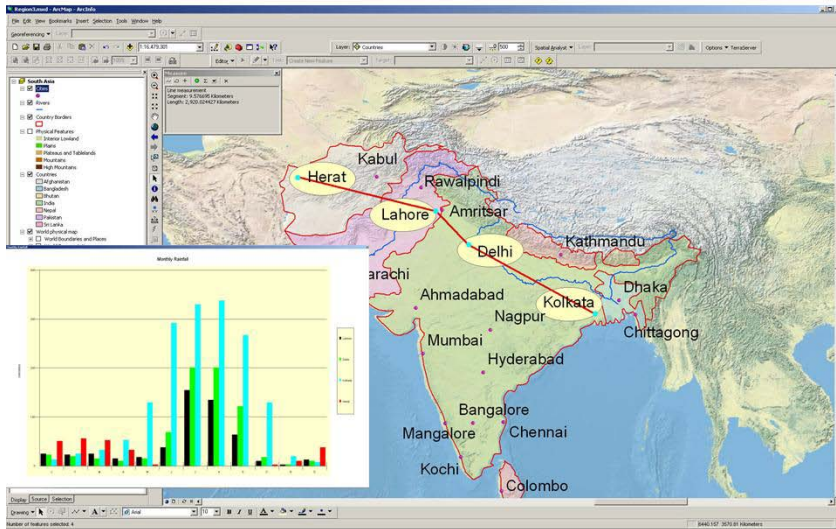
Each of these lessons connects to the following NCTM **Content** standards: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis. Each of these lessons connects to the following NCTM **Process** standards: Problem solving, reasoning and proof, communication, connections to other disciplines (in particular, geography, economics, and Earth Science), and representation. In all units, the content and skills is connected to other areas of mathematics, to other school subjects, and to real-world applications. GIS is by nature a systems and holistic approach to examining the world. Students organize, record, and communicate information and ideas using words, maps, tables, and symbols. They learn to interpret information presented in various forms.

Why Use GIS for Mathematics Instruction?

GIS provides an excellent way to teach mathematical concepts and skills. The value of visualizing numbers is affirmed throughout the US Principles and Standards for School Mathematics, designed by the National Council of Teachers of Mathematics (NCTM). Representing numbers, understanding patterns,



relationships, and function, 2-D and 3-D geometric and spatial relationships, probability, statistics, change, models, measurements, problem solving, reasoning, connections, and communications are critical concepts. Every one of these can be explored using GIS tools and methods. Comparing graphs and maps of birth and death rates over time and region, analyzing the response of a stream to a recent storm through a real-time hydrograph, and creating cross-sections of terrain are three common activities in geography instruction, easily done in a GIS environment.



All of them—and thousands more geographic activities—involve analyzing numbers. One might say that GIS is visualizing numbers, since its basis is representing numbers as cells, points, lines, or polygons on a map. For example, the map shown here represents the results of the analysis of rainfall seasons and amounts in South Asia using ArcGIS software.

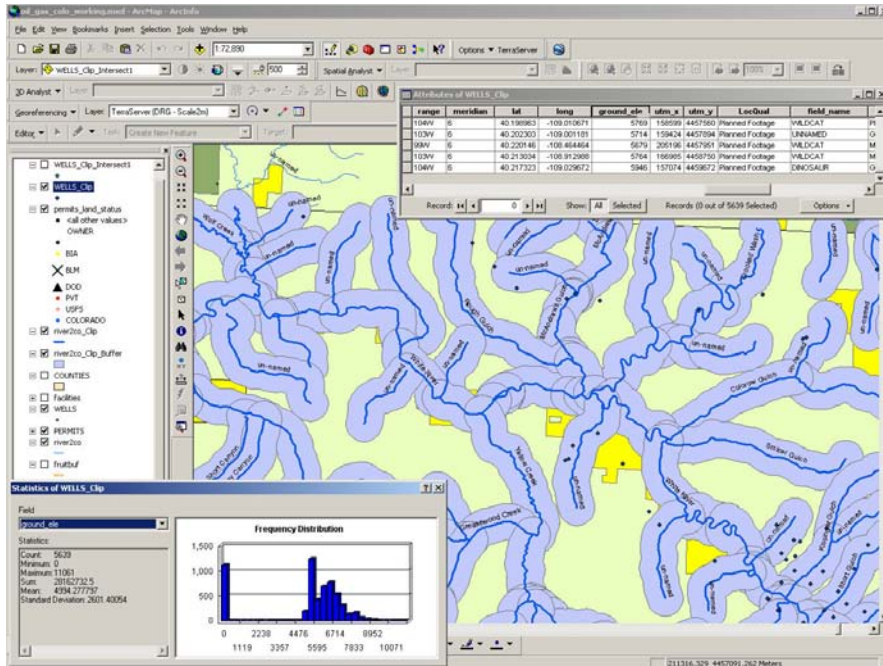
NCTM’s curricular “focal points” also connect well with GIS. A focal point must pass three rigorous tests: Is it mathematically important, both in mathematics and for use in applications in and outside of school? Does it “fit” with what is known about learning mathematics? Does it connect logically with the mathematics in earlier and later grade levels? When we connect latitude and longitude to the Cartesian coordinate system, when we measure area, shape, size, and distance in different map projections, when we compare geometric to exponential growth rates of agricultural output, even when we explain the Earth’s shape, rotation, and revolution, we are applying geographic and mathematical concepts and can use GIS to teach it.

Let’s get more specific. First, the use of GIS brings math to life by making math visual. Think of common problems such as: “Where and when will these trains



cross paths? One departs Point A at 6:00am and heads toward Point B at 70kph, while the other departs Point B at 6:30am bound for Point A at 60kph.” Where and when will they cross paths?” GIS allows the anchoring of these and other problems in the real world: Points A and B could be Cheyenne and Casper, and students can determine and plot the course and passing point at a real point on a GIS map layer.

Second, solving math problems in a GIS environment allows students to grapple with biodiversity, crime, natural hazards, climate, energy, water, and other relevant real-world issues of the 21st Century. For example, the map below shows the analysis of the elevation of oil and gas wells in western Colorado.



Third, students often do not feel that what they are learning is relevant to what they will be doing after they get out of school. Hundreds of jobs in geospatial technologies—not just surveyors and remote sensing analysts—require analytical, statistical, and computational skills that are learned in mathematics.

To build bridges with mathematics means to serve with primary, secondary, and university mathematics educators on advisory boards at all levels, to conduct professional development with them and for them, and to regularly dialog with teachers and administrators about the clear linkages between GIS and mathematics. Curricular materials that incorporate math concepts and skills are critically needed and can be best created through partnering with math teachers and professors.



The use of GIS supports all of the key NCTM standards featured on the NCTM illuminations site (<http://illuminations.nctm.org/Standards.aspx>). These include numbers and operations, algebra, geometry, measurement, data analysis and probability. Specifically, these include working with vectors (direction and length), perimeter and area, working with cell-based raster data, working on estimation, units of measure, shapes and patterns, speed and distance, modeling, percentages, and ratios.

These connections have deep historical roots: Willis Ernest Johnson wrote *Mathematical Geography* back in 1908, and Eratosthenes connected geography to mathematics in measuring the Earth's circumference over 2,200 years ago! I encourage you to enrich the teaching and learning of mathematics with critical thinking and spatial analysis through the use of GIS technology.

National Governors Association and CCSSO Common Core Standards

The National Governors Association Center for Best Practices and the Council of Chief State School Officers in 2009 began releasing common core standards for public comment. Beginning with career and workforce-readiness standards, it continued in 2010 with the release of English Language Arts and Mathematics standards. As these standards are aligned with college and work expectations, apply higher-order skills, and especially because they all seem to revolve around problem-solving, a strong case can be made through the common core initiative why teaching with the spatial perspective is an essential part of primary, secondary, higher, and informal education. Furthermore, the math core content standards include such items as attending to precision, and constructing viable arguments, both of which are core to GIS.

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They break things down into cases and can recognize and use counterexamples. They use logic to justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. They make sense of complex problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They consider analogous problems, try special cases and work on simpler forms. They evaluate their progress and change course if necessary. They try putting algebraic expressions into different forms or try changing the viewing window on their calculator to get the information they need. They look for correspondences between equations, verbal descriptions, tables, and graphs. They draw diagrams of relationships, graph



data, search for regularity and trends, and construct mathematical models. They check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They look for and make use of structure. Mathematically proficient students look closely to discern a pattern, and GIS is all about investigating patterns. They make strategic decisions about the use of technological tools. GIS is a technological tool but one that encourages critical thinking about data and processes.

In terms of statistics, GIS offers many connections. With GIS, data are gathered, displayed, summarized, examined and interpreted to discover patterns. Data can be summarized by a statistics involving measuring center, such as mean and median, and statistics measuring spread, such as interquartile range and standard deviation. Different distributions can be compared numerically and visually, using different classification methods, histograms, and standard deviational ellipses. Which statistical measures to use, the results of these considerations, depend on the problem investigated, the data gathered and considered, and the real-life actions to be taken.

We encourage you to consider using spatial analysis with GIS in your mathematics instruction.

